CS-BIGS 4(2): 93-100 http://legacy.bentley.edu/csbigs/documents/camiz.pdf

Procrustes Analysis and Stock Markets

Sergio Camiz
Sapienza Università di Roma, Italy

Jean Jacques Denimal
Université des Sciences et Technologies de Lille, France

In this paper, Procrustes Analyses are introduced with their various facets and set in an economics framework with an application to stock markets. The time series of the daily indexes of eight major world stock markets are considered for a four-year period (1988-91) and five two-year mobile windows are built to find a common graphical representation in which the evolution of the relations among the stock markets can be appreciated. Both Procrustes and Generalized Procrustes Analyses are applied in order to illuminate this evolution.

Introduction

In ancient Greek mythology, the bandit Procrustes ("the Stretcher") was a son of Poseidon with a stronghold on Mount Korydallos, situated on the sacred way between Athens and Eleusis. There, he had an iron bed in which he invited every passer-by to spend the night. According to the guest's size with respect to the bed, he would either stretch him with his smith's hammer to fit the bed, or amputate the excess length. Indeed, nobody ever fit the bed exactly because Procrustes would take advantage of two beds which differed in size. Procrustes continued his reign of terror until Theseus, travelling to Athens along the sacred way, "fitted" him to his own bed (Plutarch).

In data analysis, the *Procrustes Analysis* (*PA*) is a method that provides the best adjustment of a set of points, called *test cloud*, to a given set, called *target cloud*, according to transformations that do not change, up to a scale factor, the reciprocal distances among the points of the test cloud. Originally proposed by Mosier (1939), its name is due to Hurley and Cattel (1962), and it later underwent further developments, in particular the *Generalized Procrustes Analysis* (*GPA*, Gower, 1975), in which a best adjustment is searched among several sets of points. The method is largely used both in pattern recognition and in the so-called *shape analysis* (Dryden and Mardia, 1998) as a first adjustment of more complex transformations, but it may be applied to all situations in which direct comparisons among configurations of the same objects under different representations are requested.

The exploratory analysis of time-series through classical methods may be performed by considering the observations as units and the occasions of observation as variables: its drawback is the very large number of variables in respect of the usually smaller number of units. Bry (1995) proposes to transpose the data table, so that the series represent the variables and the occasions the units. In this way, classical scaling methods, such as *Principal Component Analysis* (*PCA*, Bry, 1995) and its variations, may be adopted. This allows to investigate the overall relations among time-series, based on their correlation matrix, and to detect the influence of the occasions as displayed by their position on the *PCA* principal axes and planes. If some further information is searched, in particular to identify specific time-periods during which the pattern of correlations may be different from the overall one, mobile windows on which to apply the same analyses may be adopted. Camiz et al. (2010) applied this method to a set of yearly time-series of tree-ring width of *Notofagus* trees of Tierra del Fuego (Argentina), resulting in a time-series of *PCA*, whose synthesis seemed difficult. The application of Procustes Analysis to this case arose as a possible solution.

In this paper, we aim at describing both *PA* and *GPA* principles and we show their application to time-series through an example: the evolution of eight international stock-exchange markets, as represented by five tables of daily observations for two years. For this task a pre-treatment is needed, in order to get a set of points in the space corresponding to the series; for our purposes, we shall use first a Non-Metric Multidimensional Scaling (*NMMDS*, Borg and Groenen, 2005), based on a distance matrix derived from the correlation between series, to obtain a graphical display representing the relations between time-series. This will provide us with the clouds of points on which *PA* may be applied.

The Procrustes Transformation

The aim of a *Procrustes Analysis* (*PA*) is to best adjust two clouds of points in a geometrical space, that is to adjust a *test cloud* to a *target cloud* as best as possible through a *rigid transformation*. As rigid transformations we consider only translations, rotations, and rescaling, or a composition of these.

Let there be in R^p two clouds of points, the target cloud N_x of *n* points x_i and the test cloud N_z of *n* points z_i indexed by the same set *I* (with $n = card(I)$), and given the same weights p_i assumed to be strictly positive and to sum up to 1. In addition, we assume that R^p is given a metric represented by a positive definite matrix M. This is the most general framework: normally, all points have the same weight and the metric M is given by the identity matrix *I*.

The *PA* consists in finding the so-called *Procrustes Transformation* (*PT*) *P(a,T,s)*, composed of a translation a, $a \in R^p$, a rotation *T*, $T \in R^{p \times p}$, and a rescaling *s*, $s \in R$, such that the images of the points of the test cloud N_z under *P*, that is $u_i = P(z_i) = s.T.z_i + a$, $i \in I$ are as close as possible to the points of the target cloud N_x in the least-squares sense. The objective function to minimize is thus: $PA = \sum_{i \in I} p_i . \|x_i - u_i\|_M^2$, with the norm $\| \; \|_M$ depending on the metric M, a symmetric definite positive matrix.

Search for the translation a

If we introduce both clouds centroids $G = \sum_{i=1}^{n} p_i . x_i$ and $H = \sum_{i=1}^{n} p_i . z_i$, it is easy to see that the transformation of H, $U = P(H) = s.T.H + a$ minimizes *PA* when $G = U$, which implies: $a = G - s.T.H$. Indeed, from the objective function it follows that:

$$\sum_{i \in I} p_i . \|x_i - u_i\|^2 = \sum_{i \in I} p_i . \|x_i - G\|^2 + \sum_{i \in I} p_i . \|u_i - U\|^2 \\ - 2 . \sum_{i=1}^{n} p_i . \langle x_i - G, u_i - U \rangle + \|G - U\|^2$$

where the only member depending on *a* is $\|G - U\|^2$, hence its minimum for $G = U$.

Search for the rescaling s

If we replace the found translation a into the objective function, this becomes

$$s^2 . \left[\sum_{i=1}^{n} p_i . \|T.(z_i - H)\|^2 \right] - 2.s. \sum_{i=1}^{n} p_i . \langle x_i - G, T.(z_i - H) \rangle$$

whose minimum is reached when its derivative is zero, that is

$$2s. \left[\sum_{i=1}^{n} p_i . \|T.(z_i - H)\|^2 \right] \\ - 2 \sum_{i=1}^{n} p_i . \langle x_i - G, T.(z_i - H) \rangle = 0$$

hence

$$s = \frac{\sum_{i=1}^{n} p_i . \langle x_i - G, T.(z_i - H) \rangle}{\sum_{i=1}^{n} p_i . \|T.(z_i - H)\|^2}$$

Since any rotation is a norm-preserving isometry, T does not influence the denominator and may be removed, giving

$$s=\frac{\sum_{i=1}^{n} p_i.\langle x_i-G,T.(z_i-H)\rangle}{\sum_{i=1}^{n} p_i.\|(z_i-H)\|^2}$$

Search for the rotation T

Once the found rescaling *s* is plugged into the objective function, it becomes

$$\sum_{i\in I} p_i.\|x_i-u_i\|^2=\sum_{i\in I} p_i.\|x_i-G\|^2+\frac{\left[\sum_{i=1}^{n} p_i.\langle x_i-G,T.(z_i-H)\rangle\right]^2}{\sum_{i=1}^{n} p_i.\|z_i-H\|^2}-2.\frac{\left[\sum_{i=1}^{n} p_i.\langle x_i-G,T.(z_i-H)\rangle\right]^2}{\sum_{i=1}^{n} p_i.\|z_i-H\|^2},$$

simplifying to

$$\sum_{i\in I} p_i.\|x_i-u_i\|^2=\sum_{i\in I} p_i.\|x_i-G\|^2-\frac{\left[\sum_{i=1}^{n} p_i.\langle x_i-G,T.(z_i-H)\rangle\right]^2}{\sum_{i=1}^{n} p_i.\|z_i-H\|^2}.$$

If we center both vectors x_i and z_i to their respective centroids, say $\tilde{x}_i=x_i-G$ and $\tilde{z}_i=z_i-H$, the search for the rotation T is reduced to the maximization of

$$\sum_{i=1}^{n} p_i.\langle \tilde{x}_i,T.\tilde{z}_i\rangle .$$

Let us now define two matrices $V=\sum_{i=1}^{n} p_i.\tilde{x}_i.{}^t(\tilde{z}_i)$ and $W=V.M.{}^tV.M$, with M the metrics in R^p: clearly MW is symmetrical (so that W will be called M-symmetrical). Thus, W admits an M-orthonormal basis of eigenvectors, denoted $\{e_1,e_2,\ldots,e_p\}$ that we assume sorted in decreasing order of their corresponding eigenvalues $\lambda_1\ge\lambda_2\ge\ldots\ge\lambda_p$ (Golub and van Loan, 1996). If the first r eigenvalues are different from zero, we can define M-orthonormal vectors $f_\alpha=\frac{{}^tV.M.e_\alpha}{\sqrt{\lambda_\alpha}}\in R^p$, $\alpha=1,..,r$, that may be completed to an M-orthonormal basis $\{f_1,f_2,\ldots,f_p\}$ of R^p. Indeed,

$$\langle f_\alpha,f_\beta\rangle={}^tf_\alpha.M.f_\beta=\frac{{}^te_\alpha.M.V.M.{}^tV.M.e_\beta}{\sqrt{\lambda_\alpha.\lambda_\beta}}=\begin{cases}0 & \text{if } \alpha\ne\beta\\ 1 & \text{else}\end{cases}.$$

Bourgois (1978) showed that a possible T that gives the searched optimum is defined by $T.f_\alpha=e_\alpha$, $\forall\alpha\in[1,r]$.

Remark 1

Let X and Z be two tables with dimensions $[n,p]$ whose rows correspond to the two clouds N_x and N_z respectively. When the metric M of R^p is the classical Euclidean one and the weights $\{p_i / i\in I\}$ are all equal, then the three steps of the transformation, say the search for the rotation *T*, the translation *a*, and the rescaling *s* may be described as (see Borg and Groenen, 2005):

1) Centering the tables $\tilde{X}$ and $\tilde{Z}$,
2) Compute the tables product[1] $C={}^t\tilde{X}.\tilde{Z}$,
3) Compute the Singular Value Decomposition of $C=P.\Lambda.{}^tQ$,

with the following results:

- the rotation $T=P.{}^tQ$
- the rescaling $s=\frac{trace({}^t\tilde{X}.\tilde{Z}.{}^tT)}{trace({}^t\tilde{Z}.\tilde{Z})}$
- the translation $a=\frac{1}{n}.{}^t(X-s.Y.{}^tT).1_n$,

with 1_n a vector with all components equal to 1.

Remark 2

The objective function $PA=\sum_{i\in I} p_i.\|x_i-u_i\|^2$, with $u_i=s.T.z_i+a$ may be reformulated as

$$Trace\left[{}^t\left(X-(s.Z.{}^tT+1_n.{}^ta)\right)\left(X-(s.Z.{}^tT+1_n.{}^ta)\right)\right]$$

[1] In this paper, we shall denote by ${}^tX$ the transpose of matrix X.

This formulation allows one to generalize this criterion, as will be discussed below.

Analysis of synchronous time-series evolution

Let us consider a set of p synchronous time-series observed along n occasions: they may be gathered in an $n \times p$ data table S and their overall relations may be described by their correlation matrix R. If we want to study the evolution of the system, we may want to consider their correlation in different time-intervals and compare them. This may be done by defining a *mobile window*, a sub-table of X of fixed length w, and by shifting it along the original table. In this way, we would obtain a set of indexed $w \times p$ tables S_k whose structures, represented by the corresponding correlation matrices R_k, may be compared. To perform this comparison, we can build Euclidean representations of each table and compare them, if possible with a simultaneous representation.

Indeed, given any dissimilarity δ, that is a symmetric non negative real function of a couple of points with $\delta_{ii} = 0$, such a Euclidean representation may be obtained through *NMMDS* (Non-Metric Multi-Dimensional Scaling, Borg and Groenen, 2005) techniques. In the given case, we can define as dissimilarity $\delta_{ij} = 1 - r_{ij}$, the correlation between two series: δ_{ij} ranges from 0, in the case of r = 1, that is total positive correlation, to 2 when r = -1, corresponding to total negative correlation.

The goal of *NMMDS* is, given a set of p points with any dissimilarity ($p \times p$) matrix $\Delta = (\delta_{ij}),\ i, j = 1,\ldots,n$ between them, to build a matrix X ($p \times q$) composed by the coordinates of the p points in a q-dimensional Euclidean space ($q < p$), so that the so-called *stress* function is minimized, namely:

$$Stress = \frac{\sum_{i=1}^{p} \sum_{\substack{j=1 \\ i<j}}^{p} \left[(1 - r_{ij}) - d_{ij}(X) \right]^2}{\sum_{i=1}^{p} \sum_{\substack{j=1 \\ i<j}}^{p} (1 - r_{ij})^2}$$

in which $d_{ij}(X)$ is the Euclidean distance between points, computed from the coordinates in X.

The minimum stress may not be found algebraically but through the use of some specific numerical algorithm, as for example the *SMACOF* (Scaling by MAjorizing a COmplicated Function, De Leeuw, 1988). It must be pointed out that, unlike in metric scaling (such as *PCA*), the Euclidean representation resulting from *NMMDS* is not unique, since any isometry does not modify the stress. To choose a suitable dimension q of representation, the so-called *elbow criterion* may be used: it is the rule of thumb that consists in examining the scatter plot of the stresses as a function of q and choosing as suitable the dimension in which an elbow occurs (Thorndike, 1953).

Thus, if X_k and X_{k+1} are two representations associated to two time periods, a Procrustes Transformation will be searched in order to get X_k and $s.X_{k+1}.T + a$ to be as close as possible. Indeed, in this case we would not consider both a rescaling s that would modify the metrics of one table, and a translation a, since both X_k and X_{k+1} are column centered. Thus, the searched transformation will be the transformation T minimizing $\left\| X_k - X_{k+1}.T \right\|^2$.

Application to stock exchange markets

We propose here an application, analogous to that of Groenen and Franses (2000), concerning 3347 daily overall indices of 8 great stock exchanges during a period ranging from January 2nd 1986 to October 29th 1998. The data were taken from Franses and van Dyck (2000) and downloaded from http: //robjhyndman.com /tsdldata/data/FVD1.dat. All computations were performed through the *SAS statistical software*.

We consider here two periods of two years 1988-1989 and 1990-1991. For each of these two periods, an 8 by 8 correlation matrix $R_k = (r_{ij}), i \in [1,8], j \in [1,8], k \in [1,2]$ between indices is computed based on the daily values. Our goal is to represent the stock exchanges as points in the Euclidean multidimensional space based on the correlation matrix, so that the closeness of two points represents a high positive correlation between corresponding stock exchanges.

In our case n = 8 and the elbow criterion suggests q=2 for both cases, so that the representations will be 2-dimensional, in other words a plane. In Figures 1 and 2, the plane representations obtained through the *NMMDS* applied to the two chosen periods are shown.

Comparing the two graphics, the isolation of *Hong-Kong* is evident in 1988-89, whereas in 1990-91 both *New York* and *London* approach *Hong-Kong* in contrast with the other stock markets. In Figure 3, the two graphics are superimposed, according to a Procrustes Transformation, namely an isometry: with this representation even the intensity of the variation of each stock market may be

Figure 1. Representation of the 8 stock market correlations in 1988-1989.

Figure 2. Representation of the 8 stock market correlations in 1990-1991.

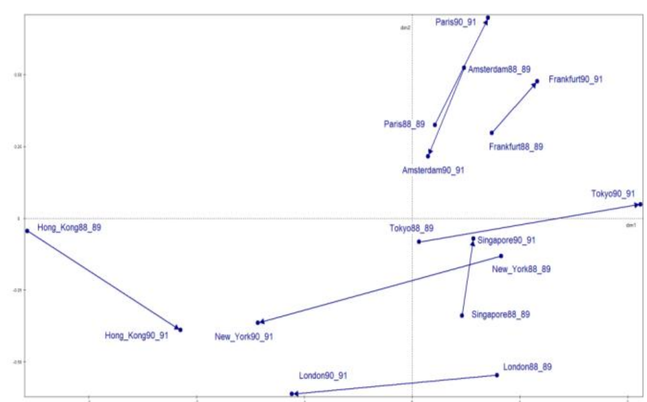

Figure 3. Procrustes simultaneous representation of the 8 stock markets.

appreciated, paying attention to the directions of the arrows in the figure.

Looking at Figure 3, we may now say that not only did *New York* and *London* approach *Hong-Kong* in the second window, but also that *Hong-Kong* approached these stock markets, whereas *Tokyo* moved further, as did *Paris* and *Frankfurt*. Morevoer, a convergence of *Amsterdam* with *Singapore* may be appreciated.

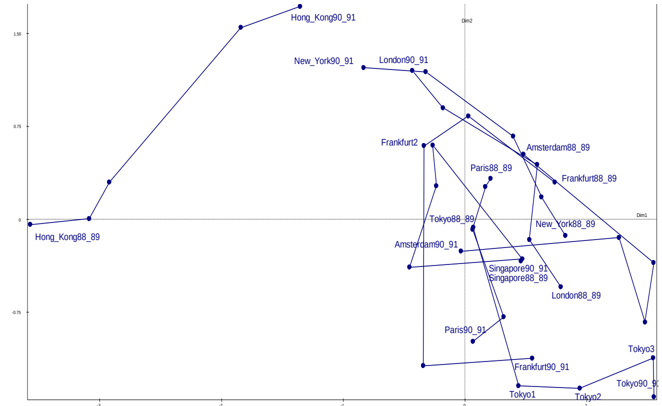

Figure 4. Representation of the evolution of the 8 stock markets by successive Procrustes transformations.

Now, in order to fine-tune the evolution of the correlations among the stock exchanges during these years, we build a series of mobile windows each two-years long, with a shift of 6 months between two adjacent windows: thus, three intermediate patterns result and we want to represent the resulting five *NMMDS* graphics simultaneously. For this task, a *PT* is applied between two successive representations, to adjust each representation on the preceding one.

In Figure 4 the evolution of the correlations between stock markets during the period 1988-1991 obtained in this way is shown.

Looking at the graphics, the regular evolution of *Hong-Kong*, *London*, *New York*, and *Paris* may be appreciated, whereas the pattern for the other markets is much more complicated.

Indeed, this last application involving five tables, may be alternatively dealt with directly, by searching for a transformation that simultaneously best adjusts each of *K* configurations to all others: this will be shown in the following section.

The Generalized Procrustes Analysis

The idea of *Generalized Procrustes Analysis* (*GPA*, Gower, 1975) is to simultaneously best adjust a set of *K* clouds of points in a geometrical space through a rigid transformation, again composed only by translations, rotations, and rescaling. Indeed, this is an alternative to the successive *PT*s applied to the last example of Section 4, with the advantage that in this case a *compromise* can be built, that is a graphical representation of only one cloud that approaches at the best all the given ones.

Given K data tables $X_1, X_2, \ldots, X_K$, all with dimensions (n, p), the GPA aims at minimizing the following criterion, that generalizes the notation of remark 2:

$$GPA = \sum_{k=1}^{K} \sum_{\substack{l=1 \\ k<l}}^{K} trace\left[{}^t\left(\tilde{X}_k - \tilde{X}_l\right)\left(\tilde{X}_k - \tilde{X}_l\right) \right]$$

where $\tilde{X}_k = s_k . X_k . T_k + 1_n . {}^t a_k$, $\forall k \in [1, K]$.

This criterion may be minimized only iteratively, because no analytical solution is known to date. In the sequel, three methods of minimization of GPA are described.

First method

The first method, proposed by Kristof and Wingersky (1971), results in a series of adjustments of one configuration, considering all others fixed; thus, iteratively, an adjusted $\tilde{X}_1$ assuming the others fixed is searched for, then an adjusted $\tilde{X}_2$ assuming the others fixed is searched for, and so on.

More precisely, for every configuration $k \in K$, the GPA criterion may be written as:

$$GPA = (K-1) . trace({}^t\tilde{X}_k . \tilde{X}_k) - 2 . trace({}^t\tilde{X}_k . \sum_{l \neq k} \tilde{X}_l) + C$$

where C represents the terms of GPA that do not depend on $\tilde{X}_k$.

By setting $Y = \frac{1}{(K-1)} . \sum_{l \neq k} \tilde{X}_l$, it follows that:

$$GPA = (K-1) . \left[trace({}^t\tilde{X}_k . \tilde{X}_k) - 2 . trace({}^t\tilde{X}_k . Y) \right] + C,$$

and therefore that

$$GPA = (K-1) trace\left[{}^t\left(\tilde{X}_k - Y\right) . \left(\tilde{X}_k - Y\right) \right] + \left[C - (K-1) trace({}^tY . Y) \right].$$

This corresponds to minimizing $trace\left[{}^t\left(\tilde{X}_k - Y\right)\left(\tilde{X}_k - Y\right) \right]$, that is to apply the Procrustes transformation of the test configuration X_k on the target Y, as previously described. The convergence is reached with few iterations, since the GPA criterion is positive and decreases at every step.

Second method

The second method for minimizing the GPA criterion was proposed by Gower (1975) and improved by Ten Berge (1977).

As a first step, the K translations a_k are found: it may be shown that the GPA criterion is minimized if all resulting configurations have the same centroid: $\forall k \in [1, K]$, ${}^t\tilde{X}_k . 1_n = 0$. This is obtained by simply centering the original tables X_k to their respective centroid.

Both rotations T_k and rescalings s_k are then found iteratively in two phases, once we set initially: $s_k = 1$, $\forall k \in [1, K]$ and $\tilde{X}_k = X_k$:

1) In the same way as in the first method, a rotation T_k is searched to adjust $\tilde{X}_k$ on $Y = \frac{1}{(K-1)} . \sum_{l \neq k} \tilde{X}_l$ iteratively for every k until convergence. The obtained configurations are again denoted by $\tilde{X}_k$, $k \in [1, K]$.

2) The $K \times K$ matrix B is now considered, whose elements are $b_{kl} = trace\left({}^t\tilde{X}_k . \tilde{X}_l\right)$ and we denote by $\Phi = (\phi_k)_{k \in [1,K]}$ the eigenvector of B associated to its largest eigenvalue. It may be shown that the rescaling that minimizes the GPA criterion is
$$s_k = \frac{\sum_{l=1}^{K} trace({}^t\tilde{X}_l . \tilde{X}_l)}{trace({}^t\tilde{X}_k . \tilde{X}_k)} . \phi_k,$$ see Ten Berge (1977).

Again, we denote by $\tilde{X}_k$, $k \in [1, K]$ the rescaled configurations.

Phases 1) and 2) are then repeated until convergence.

Third method

A third way to minimize the GPA criterion (see Borg and Groenen, 2000) is based on the average configuration $Z = \frac{1}{K} . \sum_{k=1}^{K} \tilde{X}_k$. Indeed, the criterion may be written as

$$GPA = K . \sum_{k=1}^{K} trace\left[{}^t\left(\tilde{X}_k - Z\right)\left(\tilde{X}_k - Z\right) \right]$$

The procedure minimizes the *GPA* criterion according to the following steps:

1) The configurations $\tilde{X}_k$ are found by a *PA* of the k test configurations on the average configuration Z assumed fixed.

2) The average configuration $Z = \frac{1}{K} . \sum_{k=1}^{K} \tilde{X}_k$ is re-calculated.

Steps 1) and 2) are iterated until convergence. Albeit very simple, this method is the most time-consuming, so that its use is not recommended.

It is worth observing that, in all methods, the average configuration $Z = \frac{1}{K} . \sum_{k=1}^{K} \tilde{X}_k$ results in a compromise among all configurations, that summarizes the K clouds in a single one.

Application of GPA to stock exchange markets

The application of *GPA* to the eight stock exchange markets results in two graphics: a compromise (Figure 5) and a representation of the evolution along time (Figure 6). All computations were performed with the New*MDSX* package.

In Figure 5 the compromise representation of the eight stock markets with respect to the five different periods is given as a centroid cloud. It is clear that *Hong-Kong* is different from the other markets through all the windows, and that some difference concerns both *Tokyo* and *Amsterdam.*

In Figure 6 the evolution of the stock markets is shown, as resulting by the application of *GPA*. The pattern is somehow different from the one shown in Figure 4. This clearly is due to the different criterion used in the analysis, since the previous one adjusted every table to its predecessor and the latter did it to the common centroid. Nevertheless, the main evolution is the same in both representations: the convergence of *Hong-Kong, New York,* and *London,* the divergence of both *Tokyo* and *Frankfurt* with respect to these, but eventually evolving in the same direction.

Conclusion

In this paper we introduced both *PA* and *GPA* in their classical formulation and suggested a possible application to stock market time series. However, other techniques

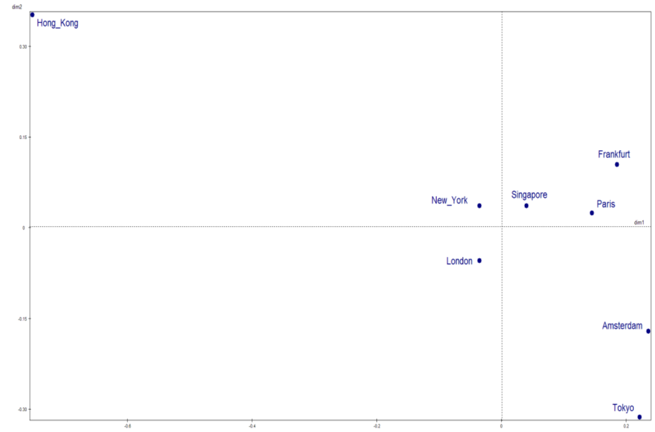

Figure 5. GPA of the eight stock exchange markets. Compromise representation given by the centroid.

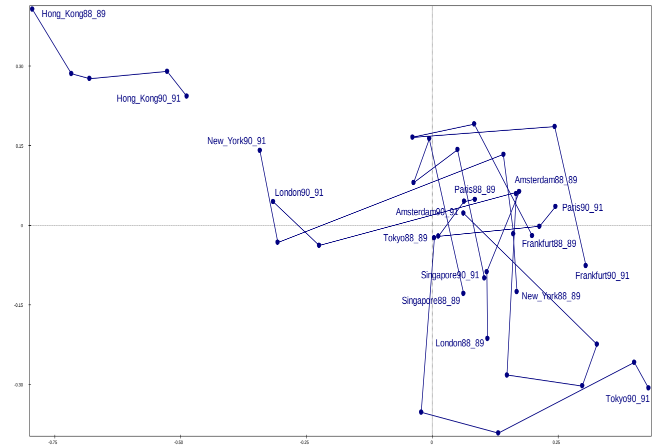

Figure 6. Representation of the evolution of the 8 stock markets by the Generalized Procrustes Analysis.

for the same purpose may be taken into account, based on different rationale and a comparison of the results may be advisable: we can quote here 3-way methods, both metric, such as *Dual Statis* (Lavit, 1988) and *Dual Multiple Factor Analysis* (Lê *et al.*, 2008), and non-metric, such as *INDSCAL* (Carroll and Chang, 1970).

Generalizations of Procrustes Analysis are discussed in Borg and Groenen (2005) and implemented in both *PINDIS* (Lingoes and Borg, 1978) and New*MDSX*. Further developments may also be considered, in particular when special conditions occur. When the series need to be considered with different weights, depending on either the market's importance or on the different range of variation of the markets in the different occasions or on their correlation, the least-squares approach is not adequate since it gives the same importance to all involved markets. Thus, a more general model may be considered, such as Maximum Likelihood (Theobald and Wuttke, 2006). When the samples are different from an occasion to another, new *adaptive* methods may be considered. With these methods, *stochastic links* are created, as proposed by both Bouveyron and Jacques (2010) and Bienacki and Lourme (2010), may be applied. The stochastic links may be seen as a generalization of the geometric transformations used in Procrustes Analysis, with the further possibility of classification or prediction otherwise impossible.

Acknowledgements

This paper was developed during the reciprocal visits of the authors in the framework of the Socrates/Erasmus agreement between Sapienza Università di Roma and the Université des Sciences et Technologies de Lille. The first author was also supported by the Facoltà di Architettura Valle Giulia of belonging. All institution grants are gratefully acknowledged.

REFERENCES

Biernacki, C. & Lourme, A. 2010. Simultaneous Mixture Model-Based Clustering of Samples from Multiple Origin. MASH-2010, Lille, *Session on Adaptive Methods.*

Borg, I. & Groenen, P.J.F. 2005. Modern Multidimensional Scaling. Berlin, *Springer Series in Statistics.*

Bourgois, P. 1978. Recherche du déplacement minimisant la distance entre deux ensembles de points homologues situés dans un même espace euclidien. *Les cahiers de l'analyse de données,* 3(4):440-448.

Bouveyron, C. & Jacques, C. 2010. Adaptive Mixture of Regressions. MASH-2010, Lille, *Session on Adaptive Methods.*

Bry, X. 1995. Analyses factorielles simples. Paris, Economica.

Camiz, S., Maulucci, R. & Roig, F.A. 2010. Exploratory analysis methods applied to dendrochronological series. In O.Papu (Ed.), Actas E-ICES - Malargüe (Mendoza, Argentina) 24-27, ottobre 2009, Mendoza, *Comisión Nacional de Energia Atómica,* 61-67.

Carroll, J.D., & Chang, J.J. 1970. Analysis of individual differences in multidimensional scaling via an N-way generalization of "Eckart-Young" decomposition. *Psychometrika,* 35:283-319.

De Leeuw, J. 1988. Convergence of the majorization Method for multidimensional scaling. *Journal of Classification,* 5:163-180.

Dryden, I.L., & Mardia, K.V. 1998. Statistical Shape Analysis. New York, *Wiley.*

Franses, P.H., & Van Dijk, D. 2000. Nonlinear Time Series Models in Empirical Finance. Cambridge, *Cambridge University Press.*

Golub, G., & Van Loan, C. 1996. Matrix Computations, third edition. London, *The Johns Hopkins University Press.*

Gower, J.C. 1975. Generalized Procrustes Analysis. *Psychometrika,* 40:33-51.

Groenen, P.J.F. & Franses, P.H. 2000. Visualizing time-varying correlations across stock markets. *Journal of Empirical Finance,* 7:155-172.

Horst, P. 1965. Factor Analysis of Data Matrices. New York, *Holt, Rinehart and Winston.*

Hurley, J. R. & Cattell, R. B. 1962. The Procrustes program: Producing direct rotation to test a hypothesized factor structure. *Behav. Sci.,* 7:258–262.

Kristof, W. & Wingersky, B. 1971. Generalization of the orthogonal Procrustes rotation procedure for more than two matrices. *In Proceedings of the 79th annual convention of the American Psychological Association*: 89-90.

Lavit, C. 1988. Analyse Conjointe de Tableaux Quantitatifs. Paris, Masson.

Lê, S., Josse, J. & Husson, F. 2008. FactoMineR: An R Package for Multivariate Analysis. *Journal of Statistical Software,* 25(1):1-18.

Lingoes, J.C. & Borg, I. 1978. A direct approach to individual differences scaling using increasingly complex transformations. *Psychometrika,* 43:491-519.

NewMDSX. A resource base for dimensional Scaling. *Sigma Essex research.* Online.http://www.newmdsx.com/

Mosier, C.I. 1939. Determining a simple structure when loadings for certain tests are known. *Psychometrika,* 4:149-162.

SAS. *SAS Institute Inc.* Online at http://www.sas.com.

Correspondence: Jean-Jacques.Denimal@univ-lille1.fr