SECONDARY MATHEMATICS TEACHERS' PROFESSIONAL COMPETENCIES FOR EFFECTIVE TEACHING OF VARIABILITY-RELATED IDEAS: A JAPANESE CASE STUDY

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TITRE

Compétences professionnelles des professeurs de mathématiques du secondaire pour un enseignement efficace des idées liées à la variabilité : une étude de cas au Japon

ABSTRACT

The importance of statistics education in secondary school has been emphasized in numerous mathematics curriculum reforms carried out recently in many countries, it being noticeable that variability may arise within all the statistical objects studied in such curricula. Despite this, there have been few attempts to conceptualize or assess empirically teachers' professional competencies for teaching variability-related ideas. This article introduces a conceptual framework for examining mathematics teachers' statistical knowledge for teaching alongside teachers' beliefs and conceptions of variability, as well as a survey instrument developed based on it. Moreover, results from conducting the survey in a case-study Japanese senior high school are reported, and some implications for teaching and teacher training are discussed.

Keywords: teachers' professional competencies, statistical knowledge for teaching, teachers' beliefs, teachers' conceptions of variability.

RÉSUMÉ

L'importance de l'éducation statistique dans le secondaire a été soulignée dans les nombreuses réformes de programmes de mathématiques menées récemment dans divers pays. Il est notoire que la variabilité peut apparaître dans tous les objets statistiques étudiés dans ces programmes. Malgré cela, les tentatives de conceptualiser ou d'évaluer empiriquement les compétences professionnelles des professeurs pour l'enseignement des idées liées à la variabilité sont rares. Cet article présente un cadre conceptuel pour examiner les connaissances statistiques pour l'enseignement que possèdent les professeurs de mathématiques en plus de leurs croyances et conceptions de la variabilité. Il présente également un instrument d'enquête élaboré sur la base de ce cadre. En outre, les résultats de l'exécution de l'enquête dans une école secondaire du deuxième cycle au Japon, à titre d'étude de cas, sont brièvement présentés, et certaines implications pour l'enseignement et la formation des professeurs sont discutées.

Mots-clés : compétences professionnelles des professeurs, connaissances statistiques pour l'enseignement, croyances des professeurs, conceptions des professeurs sur la variabilité.

1 Introduction

In recent years, curricular reforms in many countries—Japan among them—have brought into the secondary school mathematics curriculum topics related to statistics (e.g., NCTM,

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2000; MEXT, 2008a, 2008b, 2009), aiming towards statistical literacy. It is noticeable that variability—a property of a statistical object which accounts for its propensity to vary or change, which is considered by several researchers not only as a fundamental concept in statistics, but also as its "raison d'être" (e.g., Shaughnessy, 2007)—may arise naturally in many different ways in such topics. Therefore, nowadays secondary school mathematics teachers must teach several variability-related ideas—such as the ones of graphical representations of data, measures of variation, distribution and sampling—, and such work demands from them specific professional competencies, without which the aims of the mathematics curriculum regarding statistics education cannot be achieved.

Döhrmann, Kaiser and Blömeke (2012) point out that "successful teaching depends on professional knowledge and teacher beliefs" (ibid., p. 327), and, with this in mind, they framed mathematics teachers' professional competencies in terms of cognitive and affective-motivational facets (Figure 1). In such framework—which is the theoretical basis of the international study Teacher Education and Development Study in Mathematics (TEDS-M)—, Döhrmann and her colleagues highlighted *content knowledge*—or *subject matter knowledge* (SMK)—and *pedagogical content knowledge* (PCK) in the cognitive facet, and teachers' professional beliefs in the affective-motivational facet, as the most fundamental traits of effective teaching and teacher education.
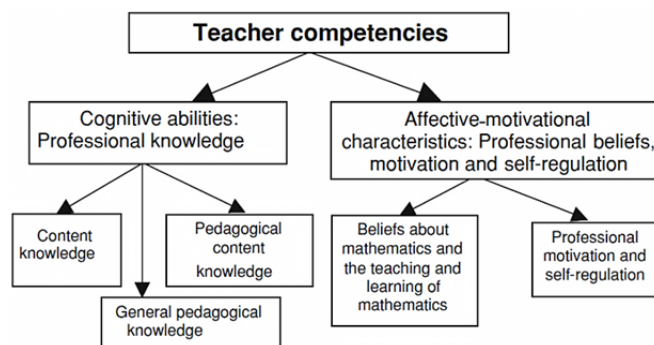

FIGURE 1 – *Conceptual model of teachers' professional competencies, according to Döhrmann et al. (2012)*

In the case of statistics education, few studies can be found in the literature focused on both the SMK and PCK entailed by teaching variability-related contents to help students achieve the aims of statistics education (e.g., Groth, 2007; Burgess, 2011; Noll, 2011), as well as on the beliefs on teaching and learning of such contents (e.g., Pierce and Chick, 2011; Eichler, 2011) and the statistical conceptions of variability (e.g., Peters, 2009; Isoda and González, 2012) held by in-service mathematics teachers. Hence, it is by no means surprising the urgent call for increasing research on these areas made by a number of concerned researchers, particularly for studies on teachers' professional knowledge and practices while teaching variability (e.g., Sánchez, da Silva and Coutinho, 2011, p. 219), as well as for teachers' beliefs on statistics itself and on what aspects of statistics should be taught in schools and how (e.g., Pierce and Chick, 2011, p. 160), and the conceptions of variability held by school mathematics teachers (Makar and Canada, 2005). Accordingly, the purpose of this paper is to respond to such calls by proposing a conceptual framework for mathematics teachers' professional competencies to teach variability-related contents, which integrates *statistical knowledge for teaching*—SKT, the professional knowledge entailed by the work of effectively teaching statistics—, conceptions of variability, and statistics-related beliefs, aiming to identify indicators that could serve to examine such traits. The proposed framework

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and indicators will serve as the base to examine the aforementioned traits in a case study group of four Japanese senior high school mathematics teachers, in order to provide a clearer picture about the level of competence to teach variability-related contents attained by them.

2 Japanese context of secondary education regarding statistics

The current Japanese national standards, known as the Course of Study (COS), were announced in March 2008 for the case of junior high school, and March 2009 for the case of senior high school. Regarding the subject of Mathematics, this COS has been fully implemented nationwide from 2012.

Following the 1998/1999 revision, which reduced by 30% the total number of class periods in the subject of Mathematics, the current COS boosted mathematics classes by 10%, and stipulated the teaching of statistics-related contents in all grades, starting from Grade 1. Then, in the case of junior high school, the COS now identifies four strands or mathematics content areas, one of which is "Practical use of data". For senior high school, the current COS also identifies four strands, one of which is "Analysis of data".

An examination of the latest COS in Japan reveals that most of the statistical contents in it are ideas related to descriptive statistics, whose purpose is mainly to handle data in order to present its salient features in an intelligible form. To that end, statistical ideas such as measures of central tendency, range, frequency distribution tables and histograms are taught in Grade 7, in order "...to foster the ability to collect data, organize data, and interpret the trend of such data" (MEXT, 2008b, p. 34). Regarding the Japanese senior high school mathematics COS for Grade 10, the overall objectives are the following:

> « To help pupils understand about … "Analysis of data", promote the acquisition of fundamental knowledge and mastery of skills, foster the ability to consider phenomena mathematically, recognize the merits of mathematics, and foster an attitude to make practical use of them. »
>
> MEXT (2009, p. 19)

In the explanation of such objectives, the Japanese COS states that "[i]n 'Analysis of data', the ideas of arithmetic mean and variability of data dealt with at junior high school are developed further, and the ideas of variance, standard deviation, scatterplot and correlation coefficient, among others, are treated". Moreover, according to the Japanese COS, "foster the ability to consider phenomena mathematically" means that "pupils are enabled to think mathematically about relationships among data such as variability and bias; explain them, by using the computer when relevant; and organize such data" (MEXT, 2009, p. 19). Also, in the remarks given about the treatment of content related to the strand "Analysis of data", it is explained that the objectives of such treatment are "[a]long with the understanding of fundamental statistical ideas, to enable students to make use of them, organize and analyze data, as well as grasp its tendency" (ibid., p. 25). Then, the objectives related to "data variability" are addressed: "Understand about the meaning of quartile deviation, variance and standard deviation, among others; make use of such ideas; grasp the tendency of data; and make explanations" (ibid., p.25). In the explanation of these objectives, it is highlighted that "[a]t the time of instruction, it is important to link terms such as quartiles, interquartile range, quartile deviation, variance and standard deviation, among others, to real events and handle them" (ibid., p. 25). As an example, the case of the boxplot aiming to compare multiple distributions is considered in some detail. In fact, it is explicitly said that "since the degree of data variability become easily recognizable in this graph, it could be used, for instance, when comparing multiple distributions of data" (ibid., p. 25).

The inclusion of the aforementioned statistical ideas in the latest COS represents a big difference with respect to its previous revision, in which all these ideas were taught at senior high school level as part of elective mathematics courses.

3 The MKT model

Ball, Thames and Phelps (2008) developed the notion of *mathematical knowledge for teaching*—henceforth MKT—, a practice-based framework focused on both what teachers do as they teach mathematics, and what knowledge and skills teachers need in order to be able to teach mathematics effectively. This model describes MKT as being made up of two domains—namely SMK and PCK—, each of them structured in a tripartite form (Figure 2).
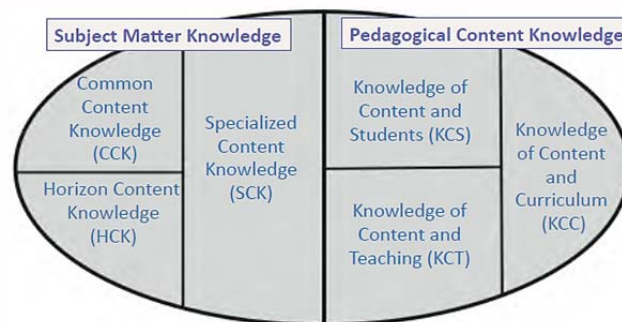

FIGURE 2 – *Domains of MKT, according to Ball et al. (2008)*

According to Ball *et al*. (2008), SMK can be divided into *common content knowledge* (CCK), *specialized content knowledge* (SCK), and *horizon content knowledge* (HCK). Furthermore, Ball and her colleagues presented a refined division of PCK, comprised of *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of content and curriculum* (KCC) (the interested reader should refer to the original article for a detailed discussion of these constructs).

Through this model, it was clarified the distinction between SMK and PCK, previous conceptualizations of such constructs were refined, and significant progress in identifying the relationship between teacher knowledge and student achievement in mathematics was made. However, as it has been highlighted by some researchers (e.g., Petrou and Goulding, 2011, p. 16), the MKT model does not acknowledge the role of either beliefs or conceptions about the subject matter in teachers' practices, which could be a drawback, since it is well documented in the literature that beliefs and conceptions are important factors influencing the work of teaching (cf. Philipp, 2007; Batanero and Díaz, 2010; Eichler, 2011; Pierce and Chick, 2011).

4 Conceptualizing teachers' professional competencies for effective teaching of variability-related ideas

While several models have been developed in the literature aiming to conceptualize MKT (cf. Petrou and Goulding, 2011), few have been done on SKT. Among them, almost all are MKT-based cognitive-oriented models—i.e., conceptualizations that have assimilated some of the categories present in the aforementioned model for MKT (cf. Groth, 2007; Burgess, 2011; Noll, 2011). Moreover, none of these MKT-based models for SKT takes into account

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all the six components identified by Ball *et al*. (2008), the role of beliefs in teachers' professional practice, or the conceptions of variability held by the teachers, which could result in an inaccurate picture of their preparedness to teach statistical contents related to variability.

In an effort to fill such gaps in the literature, an extensive literature review was carried out, and as result a conceptual model for secondary mathematics teachers' professional competencies to teach variability-related contents was proposed (Figure 3)—a detailed discussion of the conjectures that informed the development of this model can be found in González (2012). This model has two facets: one cognitive and one affective. The cognitive facet is a sixfold conceptualization of SKT, comprised of all the six cognitive constructs identified by Ball *et al*. (2008) in their MKT model, with the one of CCK—defined as the mathematical knowledge and skills expected from any well-educated adult—being adapted to meet the case of statistics. In this regard, CCK will be seen here as statistical literacy, since the acquisition of its related skills—e.g., identifying examples of a statistical concept; describing graphs, distributions, and relationships; acknowledging the omnipresence of variability in any statistical context; or interpreting the results of statistical findings and procedures—is an overarching goal of statistics education, and as such is expected from any individual after completing school education (cf. Gal, 2004; Pfannkuch and Ben-Zvi, 2011).

The affective facet of the model proposed here is comprised of two components: teachers' beliefs about statistics teaching and learning, and teachers' conceptions of variability. This is because both beliefs—defined by Philipp (2007, p. 259) as "psychologically held understandings, premises, or propositions about the world that are thought to be true"—and conceptions—the set of internal representations and the corresponding associations that a concept evokes in an individual, often explained in the literature as "conscious beliefs"—, have been widely regarded in the literature as traits that appear to influence every aspect of mathematics teaching, as well as to determine both the knowledge and beliefs concerning mathematics that students may acquire (cf. Philipp, 2007; Batanero and Díaz, 2010).
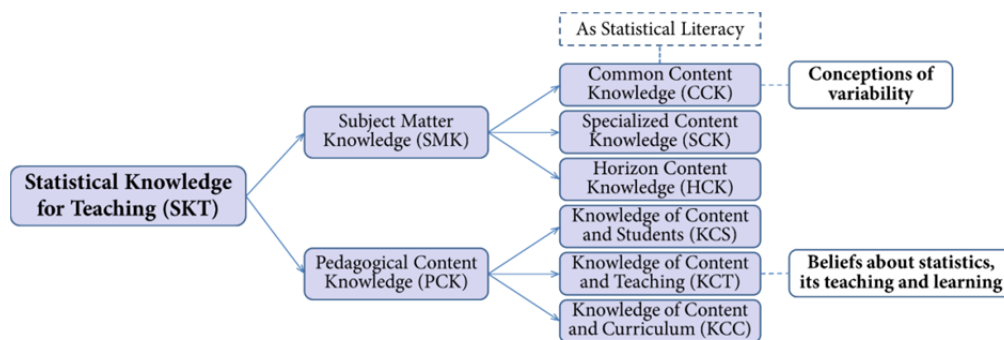

FIGURE 3 – *Proposed conceptual framework for Statistical Knowledge for Teaching (SKT)*

In order to provide a comprehensive framework for SKT, the six elements comprising it depicted in Figure 3 were paired with twelve qualitative indicators, as shown in Table 1, following a literature review and consultation with specialists. Each indicator was built from the definition of each cognitive category identified by the present study (e.g., Gal, 2004; Ball *et al.*, 2008). For example, according to Ball and Bass (2009), the so-called *horizon content knowledge* (HCK) is characterized, on one hand, by the ability to make judgments about mathematical importance from catching mathematical significance, distortions or possible precursors to future mathematical confusions or misrepresentations in what students say, and on the other hand, by the ability of building bridges between the cognitive demands of a task

with fundamental ideas, practices, values and sensibilities of the discipline. These cognitive features are acknowledged by Indicators C1 and C2, respectively (see Table 1). The rest of indicators identified in this study were developed in a similar way.

TABLE 1 – *Set of indicators proposed to assess SKT*

A: INDICATORS RELATED TO STATISTICAL LITERACY (CCK):
1. Is the teacher able to give an appropriate and correct answer to the given task?
2. Does the teacher consistently acknowledge variability and correctly interpret its meaning when answering the given task?
B: INDICATORS RELATED TO SCK:
1. Does the teacher show evidence of ability to determine the accuracy of common and non-standard arguments, methods and solutions that could be proposed to the given task by students (especially while recognizing whether a student's answer is right or not)?
2. Does the teacher show evidence of ability to analyze right and wrong solutions that could be given by students to the present task, by providing explanations about what reasoning and/or mathematical/statistical steps likely produced such responses, and why, in a clear, accurate and appropriate way?
C: INDICATORS RELATED TO HCK:
1. Does the teacher show evidence of having ability to identify whether a student response is interesting or significant, mathematically or statistically?
2. Is the teacher able to identify the significant notions, practices or values related to the statistical ideas involved in the given task?
D: INDICATORS RELATED TO KCS:
1. Is the teacher able to anticipate students' common

responses and difficulties on the given task?
2. Does the teacher show evidence of knowing the most likely reasons for students' common responses and difficulties in relation to the statistical concepts involved in the given task?
E: INDICATORS RELATED TO KCT:
1. In design of teaching, does the teacher show evidence of knowing what tasks, activities and strategies could be used to set up a productive whole-class discussion aimed at developing students' understanding of the key statistical concepts involved in the given task, instead of focusing just in computation methods or general calculation techniques?
2. Does the teacher show evidence of knowing how to sequence such tasks, activities and strategies, in order to develop students' understanding of the key statistical concepts involved in the given task?
F: INDICATORS RELATED TO KCC:
1. Does the teacher show evidence of knowing at what grade levels and content areas students are typically taught about the statistical concepts involved in the given task?
2. Does the designed lesson (or series of lessons) show evidence of teacher's knowledge and support of the educational goals and intentions of the official curriculum documents in relation to the teaching of the statistical contents present in the given task, as well as statistics in general?

5 Assessing teachers' professional competencies for effective teaching of variability-related ideas

5.1 The survey instrument

Based on the proposed framework and indicators previously outlined, as well as on the characteristics of the Japanese mathematics COS for secondary school, a pen-and-paper instrument was developed. Such instrument, designed to be completed in one hour, is based on a task addressing many variability-related ideas present in the COS, through comparing the histograms of two distributions. The chosen task was then enriched with seven SKT-related open-ended questions, aiming to elicit information about each one of the eight components of teachers' competencies to teach variability-related ideas identified by this study. There were two main arguments for using open-ended questions in this survey, being the first one that *a priori* constructed closed questions may fail to provide a set of alternatives meaningful to all respondents. The second argument was to prevent guessing or bias in the answers, since any given set of pre-coded closed alternatives might influence participants' responses.

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Each question was developed based on previous studies with similar aims reported in the literature (e.g., Meletiou-Mavrotheris and Lee, 2003; Ball *et al.*, 2008; Isoda and González, 2012). After the translation process, the resulting instrument was piloted in three stages by a total of six Japanese students of a Master's program in Educational Development, and two Japanese mathematics educators were consulted during the process. At the end of each stage, feedback was given and issues in the wording used were addressed. The piloting process allowed the author to see whether the participants would be able to understand correctly the questions, to ensure that the questions elicited the desired quantity and quality of information, and to determine if the responses would be analyzable. Moreover, from the piloting stage it was determined that respondents should require around one hour to complete the survey.

5.2   Profile of Item 1

ITEM 1

Please, read carefully the following task and answer the questions below:

Choosing the distribution with more variability. Look at the histograms of the following two distributions:
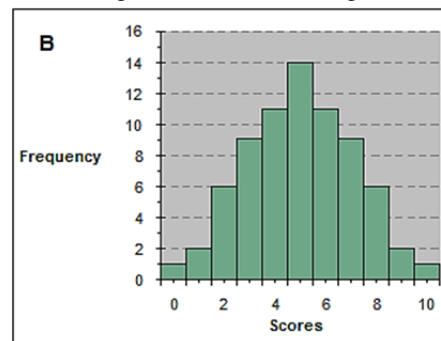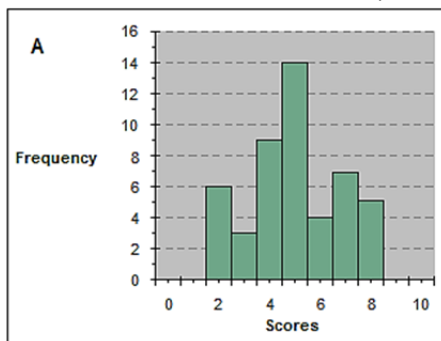
Which distribution (A or B) do you think has more variability? Briefly describe why you think this.

(a) Answer this task in as many different ways as you can. Please, be sure to show every step of your solution process.

(b) What are the important ideas that might be used to answer this task?

(c) Suppose that, after posing this task to your students, three of them come up with the following answers:

> STUDENT 1: "Distribution A has more variability because it's not symmetrical."

> STUDENT 2: "Distribution A ranges from 3 to 14, while Distribution B ranges from 1 to 14. Then, Distribution B has more variability."

> STUDENT 3: "The bars in Distribution A are clumped closer to the central bar than they are in Distribution B. Then, Distribution A has more variability."

Dealing with each student separately, please comment briefly on each of these answers, focusing on whether the answer is correct or not, why you think so, and what reasoning might have led students to produce each answer.

(d) Suppose you pose this task to your students. What are the most likely responses (correct and incorrect), and difficulties you would expect from them? Briefly explain why you think so. (Regarding to the most likely answers that you might get from the students, please do not include those mentioned in part (c).)

(e) Mathematically/statistically speaking, is any of the answers given by the students interesting or significant? If yes, briefly explain why and on what aspects. (Please, focus your response on whether there is a significant mathematical/statistical insight in the student's answer, and whether there are forthcoming contents in future classroom subjects connected to the notions being said or implied in such answer.)

(f) Briefly describe how the important ideas involved in the solving process of the given task are addressed in official curriculum documents across the different grade levels of schooling.

(g) Suppose you want to plan a lesson (or a series of lessons) to introduce the meaning of variability in the setting of the given problem to your students. Briefly describe as many instructional strategies, activities and/or tasks as you can think of that would be appropriate to use for this purpose, and sequence them accordingly, explaining why you chose to put them in such a particular order.

FIGURE 4 – *Item 1: "Choosing the distribution with more variability" task*

The final version of the survey instrument consists of a 7-question item—Item 1, which is depicted in Figure 4—, based on a task dealing with several ideas of descriptive statistics. The original version of the task in Item 1—developed by Garfield, delMas and Chance (1999), and reported in the literature as an effective means of investigating teachers' conceptions of variability in the context of histograms (e.g., González and Isoda, 2011; Isoda and González, 2012)—was modified to facilitate the calculations that could be made by the respondents. The task was also also enriched with Questions (a) to (g), aiming to elicit all the facets of teachers' professional competencies to teach statistics identified by this study. A mapping between the components of SKT that would be elicited by each question in Item 1, and the indicators associated to each cognitive trait considered by this framework, can be seen in Table 2.

TABLE 2 – *Knowledge components of SKT elicited by each of the questions posed in Item 1*

ELICITED KNOWLEDGE COMPONENT OF SKT	RELATED INDICATOR OF SKT	QUESTION
Statistical Literacy (as CCK)	A1	(a)
	A2	(a)
Specialized Content Knowledge (SCK)	B1	(c)
	B2	(c)
Horizon Content Knowledge (HCK)	C1	(e)
	C2	(b)
Knowledge of Content and Students (KCS)	D1	(d)
	D2	(d)
Knowledge of Content and Teaching (KCT)	E1	(g)
	E2	(g)
Knowledge of Content and Curriculum (KCC)	F1	(f)
	F2	(g)

The setting of the task posed in Item 1—comparing distributions—requires from teachers the mastery of several variability-related concepts—e.g., distribution, measures of variation, frequency distribution table, and histogram. Moreover, the Japanese COS for senior high school mathematics explicitly states that the setting of comparing distributions of data provides an excellent opportunity not only to study and "connect terms such as quartiles, interquartile range, quartile deviation, variance and standard deviation, among others, to real events, and handle them" (MEXT, 2009, p. 25), but also to determine the degree of data variability and map the aforementioned ideas to particular representations of data, like boxplots (ibid., p. 25). Therefore, using the Japanese COS as filter, the task in Item 1 was selected, among other things, to see how the respondents conceptualize variability, and to explore participants' base of statistical literacy. In fact, it is anticipated that teachers' answers to Question (a) will provide enough information about how teachers conceptualize variability, based on previous researches (González and Isoda, 2011; Isoda and González, 2012).

The conceptions of variability distinguished in teachers' answers to the chosen task will be classified using the eight types of conceptions identified by Shaughnessy (2007, p. 984–985), namely "Variability in particular values, including extremes or outliers", "Variability as change over time", "Variability as whole range", "Variability as the likely range of a sample", "Variability as distance or difference from some fixed point", "Variability as the sum of residuals", "Variation as covariation or association", and "Variation as distribution".

In the case of teachers' beliefs about statistics teaching and learning, the limited research on this issue suggests that they could be identified through examining the features of the lesson plans that teachers produce, such as the tasks chosen to consider a particular statistical idea, and the types of instructional strategies teachers planned to use during the lesson (Pierce

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and Chick, 2011, p. 159). What teachers planned to do while answering Question (g) will be analyzed using the four categories reflecting on teachers' beliefs developed by Eichler (2011)—i.e., traditionalists, application preparers, everyday life preparers, and structuralists—which will provide valuable information on teachers' beliefs about the nature of statistics, as well as about the teaching and learning of statistics (cf. Tatto *et al*., 2012, p. 154–156).

5.3   About the participants in this case study

In this article, it is reported a qualitative analysis of the answers to Item 1 given by four male mathematics teachers working in a public senior high school in Kure City, Hiroshima Prefecture, Japan. The respondents were between 28 and 56 years old (y.o.)—Teacher 1: 28 y.o., Teacher 2: 56 y.o., Teacher 3: 55 y.o., Teacher 4: 40 y.o.—; they had between one and thirty-four years of teaching experience (YoE)—Teacher 1: 1 YoE, Teacher 2: 34 YoE, Teacher 3: 13 YoE, Teacher 4: 19 YoE—, and were the first group of teachers that voluntarily and anonymously responded and mailed back the survey booklets, in July 2012, after having sent them by postal mail to three public secondary schools in Hiroshima Prefecture.

Participants were instructed to answer the questions with a black ballpoint pen, in the order in which they appeared, and not to go back and change answers. After the survey, no feedback was given to the participants as to correctness or appropriateness of their answers.

Since this is a case study of four mathematics teachers, generalization is not possible. However, it is anticipated that looking across the answers given by these four teachers will provide valuable insights into the current state of the SKT held by Japanese secondary school mathematics teachers, which could be suggestive of some recommendations for the educational community, as well as some policy-oriented implications.

5.4   Results and discussion regarding Question (a)

Three out of four teachers—Teachers 1, 2 and 4—answered this question. From them, two teachers used three different approaches: Teacher 1 compared the range, variance and interquartile range of both distributions; while Teacher 2 compared the range, the shape, and the mean absolute difference from the mean of both distributions. Teacher 4 answered using only one approach: by comparing the largest data span from the mean in both distributions.

It seems that all the respondents to this question made their calculations assuming both distributions to be discrete, as suggested from their use of the numerical values 2 and 8 in Distribution A, and 0 and 10 in Distribution B, as minimum and maximum values, respectively. In addition, it is worthy to highlight that the values reported by Teachers 1 and 2 of the variances, interquartile ranges, and mean absolute differences from the mean were miscalculated. Nevertheless, it is noticeable that, despite such errors, all of the interpretations made by Teachers 1 and 2 were consistent with the numerical results they obtained.

From the collected responses to Question (a), it seems that only Teacher 4 was able to give an appropriate and correct answer to the given task; thus, Indicator A1 was fully met only by Teacher 4. In relation to Indicator A2, it seems that only Teachers 1 and 4 were able to consistently identify and acknowledge variability and correctly interpret its meaning in the setting of the given task—i.e., Indicator A2 is fully met only by Teachers 1 and 4. The case of Teacher 2 is relevant to highlight, because despite him correctly using the range and the mean absolute difference from the mean of both distributions to decide on what distribution has more variability, he also used the shape to answer that Distribution A has more variability.

This wrong answer—regarding the distribution that "fluctuates the most", has "less pattern on Y", or is "less symmetrical" as the one with more variability—is not only an incorrect interpretation of the meaning of variability and an inconsistency with the other two approaches used by Teacher 2, but also represents a common misconception found in students—and, to a lesser degree, in mathematics teachers at all school levels (cf. González and Isoda, 2011; Isoda and González, 2012)—when dealing with this kind of problems (Garfield *et al*., 1999; Meletiou-Mavrotheris and Lee, 2003; Cooper and Shore, 2007).

Regarding the conceptions of variability held by the respondents, the answers given by Teachers 1 and 2 indicate they hold the eighth conception of variability identified by Shaughnessy (2007)—"Variation as distribution"—, since these teachers were able to use theoretical properties of the histograms in order to engage in transnumeration and calculate numerically—although mistakenly—some measures of variation associated to each distribution, aiming to make their decision. Although this conception implies sophisticated recognition of variability, is interesting the case of Teacher 2, who also showed evidence of holding a conception not included in the categorization described by Shaughnessy (2007), which is going to be called here "Variability as visual cues in the graph". People holding this conception regard variability as evenness (or lack thereof) in the shape of a histogram, or as closeness of fit (or lack thereof) to a normal distribution.

The answer given by Teacher 4 indicates he holds the fifth conception of variability identified by Shaughnessy (2007)—"Variability as distance or difference from some fixed point"—, since this teacher points out deviations of the endpoints from some fixed value—the mean—when asked to consider the variability in both histograms. Unlike Teachers 1 and 2, who seem to be predominantly concerned with the variability of the entire data distribution from a center, Teacher 4 does not appear to exhibit an aggregate view of data and distribution, since he seems to be rather concerned with the variability of just the endpoints from a middle.

5.5   Results and discussion regarding Question (b)

This question was posed to investigate whether teachers were able to identify fundamental statistical ideas and habits of mind significant to the correct solution of the given task. This ability is related to the so-called *horizon content knowledge* (HCK) (Ball and Bass, 2009).

TABLE 3 – *Translation of the answers to Question (b) given by the teachers in this case study*

		ANSWER GIVEN TO QUESTION (B)	
TEACHER	1	The range, the variance and the interquartile range.	
	2	N.A.	
	3	Mean	Boxplots
		Mode	Variance
		Median	Standard deviation
		Quartiles	Scatterplots
		In order to use these methods when the sample size is large, I would like to use the computer.	
	4	Because I am thinking about variability, I will focus on the maximum and minimum values of the distributions.	

Three out of four teachers—Teachers 1, 3 and 4—answered this question (Table 3). Two of them—Teachers 1 and 3—listed up multiple statistical ideas linked to the solution of the

posed task: Teacher 1 pointed out the range, the variance and the interquartile range, while Teacher 3 indicated the mean, mode, median, quartiles, boxplot, variance, standard deviation and scatterplot as important ideas required to answer the given task. Moreover, Teacher 3 indicated an important habit of mind when dealing with this kind of problems: using the computer to help with the calculations of the aforementioned statistical ideas in the case that the sample size is large. In the case of Teacher 4, he only indicated the extremes of the distributions, which are related to statistical concepts such as the range and the largest deviation from the mean. This fact introduces additional information that backs up the previous claim that Teacher 4 does not seem to exhibit an aggregate view of data.

It is relevant to notice that Teacher 1 used the same statistical ideas he identified in this question to answer Question (a). In that regard, Teacher 4 used in Question (a) the largest deviation from the mean, a statistical idea that strongly set focus on the extremes of the distribution, elements that he pointed out in Question (b). In the case of Teacher 3, despite answering this question identifying plenty of statistical ideas and an important habit of mind linked to solving the given task, he did not provide an answer to Question (a).

The responses to Question (b) seem to indicate that, besides Teacher 4—who seems to have a very limited statistical horizon—, two of the respondents—Teachers 1 and 3—were able to identify statistically significant notions related to the ideas dealt with in the given task; therefore, Indicator C2 (Table 1) seems to be fully met by these two teachers.

5.6   Results and discussion regarding Question (c)

When teachers are examining the work done by students in their absence—e.g., when marking homework papers—, they are required to have some purely mathematical way of determining the accuracy or inaccuracy of students' answers, based just on the provided numerical data or written explanations, as well as to be able to provide the likely reasoning behind students' answers. These abilities exemplify two important aspects of the cognitive construct known as *specialized content knowledge* (SCK) (Ball *et al.*, 2008; Ball and Bass, 2009), a kind of mathematical knowledge particular to the work of teaching, difficult to be articulated by other mathematically trained professionals who do not teach children.

This question was posed in order to elicit respondents' SCK in relation to the given task, by dealing with the answers to such task provided by three fictitious students. The answers given by Student 1 and 2 are examples of common misconceptions related to the estimation of variability that students frequently exhibit when comparing histograms, while the answer given by Student 3 exemplifies a wrong interpretation of a right approach to estimate variability without making calculations or engaging in the process of transnumeration.

Three out of four teachers—Teachers 1, 2 and 4—answered this question (Table 4). All these results seem to provide reliable information about Indicators B1 and B2. From their answers to Question (c), it is evident that only Teachers 1 and 4 exhibited—consistently— ability to correctly judge the accuracy of the answers given by the fictitious Students 1, 2 and 3. Therefore, Indicator B1 seems to be fully satisfied only by these two teachers. Regarding Indicator B2, from the obtained answers, it is noticeable that, again, only Teachers 1 and 4 exhibited—consistently—ability to provide clear and plausible explanations about the likely reasoning that underlies the responses given by all the fictitious students in Question (c). Thus, Indicator B2 seems to be fully met—again—only by these two teachers.

In the case of Teacher 2, the main reason that contributed to the lack of fulfilment of the SCK-related indicators appears to be his ambivalence in determining the accuracy of Student 1's answer. Moreover, Teacher 2 provided as a reason for such ambivalence that "[v]ariability cannot be determined only by means of the bilateral symmetry", which in this case is an inappropriate reason. It is worthy to mention that, by answering in this way, Teacher 2 seems to hold a common misconception about variability and symmetry commonly found in students and, in a lower degree, in mathematics teachers at all levels. This is consistent with the previous observation made in the discussion of Question (a) regarding this teacher, in which evidence of holding this misconception was clearly pointed out.

TABLE 4 – *Translation of the answers to Question (c) given by the teachers in this case study*

		ANSWERS GIVEN TO QUESTION (C)		
		LIKELY REASONING BEHIND STUDENT'S ANSWER	**CORRECT/ INCORRECT**	**REASON**
TEACHER	**1**	*STUDENT 1*: This student thinks that the bilateral symmetry of the distribution determines the degree of variability.	Incorrect	The bilateral symmetry does not have relation with the variability.
		STUDENT 2: This student thinks that the size of the range in frequency determines the degree of variability.	Incorrect	The degree of variability is expressed by the range of the data, not by the range in frequency.
		STUDENT 3: The more the data around the median, the more the variability.	Incorrect	The more the data around the median, the degree of variability becomes smaller.
	2	*STUDENT 1*: It is correct if you are thinking of normal distribution because of the bilateral symmetry, but there are distributions that, without being normal, have bilateral symmetry; given that, it cannot be said that this is correct.	?	Variability cannot be determined only by means of the bilateral symmetry.
		STUDENT 2: The larger the difference in frequency, the larger the variability.	Incorrect	Difference in frequency doesn't have relation with the variability.
		STUDENT 3: The difference between the frequencies corresponding to the score '5' and the scores '4' and '6', respectively, is larger in the case of A; therefore, Student 3 thought that Distribution A is grouped in the middle area.	Incorrect	Variability is rather smaller when data is gathered closer around the middle.
	3	N.A.		
	4	*STUDENT 1*: This student regards "bilateral symmetry" as the basis of variability.	Incorrect	"Symmetry" and variability are different.
		STUDENT 2: Variability is regarded as the difference between the highest and lowest point in frequency.	Incorrect	To think about variability we think about the difference from the mean, not about the difference in frequency.
		STUDENT 3: This student regards "variability" as the degree of data grouping in the central region.	Incorrect	In the first place, the understanding of variability is different.

5.7 Results and discussion regarding Question (d)

This question aimed at exploring teachers knowledge about students' likely responses—both correct and incorrect ones—and potential difficulties when solving the given task, as

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well as the reasons behind such responses and difficulties. By focusing on these aspects, this question intends to gain insight into teachers' *knowledge of content and students* (KCS).

TABLE 5 – *Translation of the answers to Question (d) given by the teachers in this case study*

<table>
<tr><th colspan="5">ANSWERS GIVEN TO QUESTION (D)</th></tr>
<tr><th></th><th></th><th>MOST LIKELY ANSWERS FROM STUDENTS</th><th>CORRECT/ INCORRECT</th><th>REASON</th></tr>
<tr>
<td rowspan="8">TEACHER</td>
<td rowspan="2">1</td>
<td>The comparison of the distribution ranges. Because
A: 8 – 2 = 6,   B: 10 – 0 = 10,
B could be considered to have more variability.</td>
<td>Correct</td>
<td>The range of the data is a potential way of considering the variability.</td>
</tr>
<tr>
<td>The comparison of the interquartile ranges.
A: 7 – 4 = 5,   B: 7 – 4 = 5
Since both measures are equal, the variability could be regarded as being the same.</td>
<td>Incorrect</td>
<td>By comparing only the interquartile range, it is impossible to say definitely that there is no difference in variability.</td>
</tr>
<tr>
<td rowspan="2">2</td>
<td>Distribution B has more variability from the difference between the maximum and minimum values of each data set.</td>
<td>Incorrect</td>
<td rowspan="2">When there are particular numbers such as the difference between the highest and lowest points in frequency, this "difference between maximum and minimum values" becomes larger.</td>
</tr>
<tr>
<td>Distribution A has more variability from the difference in the shape of the graphs.</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>Distribution A, because there is variability in the number of people.
The comparison is difficult because the sample sizes of A and B are different.
Distribution B varies from the scores 0~10, so it has more variability.</td>
<td></td>
<td></td>
</tr>
<tr>
<td rowspan="2">4</td>
<td>B has more variability.</td>
<td>Correct</td>
<td rowspan="2">If the difference between the maximum and minimum values of A and B is compared, this difference is larger in B.
Students might think that as the bars in B separate from the mean, they are gradually decreasing in height, but this does not apply to A, because even though its bars separate from the mean, the distribution has sections with large ones.</td>
</tr>
<tr>
<td>A has more variability.</td>
<td>Incorrect</td>
</tr>
</table>

All four teachers provided as a likely difficulty for students only one aspect of dealing with the posed task (Table 6). Teachers 1, 2 and 4 wrote down not only what they thought might be a difficulty for students when dealing with the given task, but also provided satisfactory explanations about why they considered such aspects as difficulties. In the case of Teacher 3, he wrote down one plausible difficulty that students may face while dealing with the posed task, but did not provide any reason about why he thought that way.

Regarding Indicator D2, answers to Question (d) seem to point out that such indicator is fully met—again—only by Teacher 4, the only one who consistently evidenced knowledge of the most likely reasons for the students' responses and difficulties in the given task

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TABLE 6 – *Translation of the answers to Question (d) given by the teachers in this case study*

		ANSWERS GIVEN TO QUESTION (D)	
		MOST LIKELY DIFFICULTIES FOR STUDENTS	REASON
TEACHER	1	Depending on the method of examining variability, there are different points to make judgements about it.	It might not be possible to make a right judgment about variability by using only one method.
	2	What is variability?	It is possible to make comparisons through numerical values such as the variance, but it would be difficult to make judgements about whether the data has variability or not by using only numerical values.
	3	The sample size is different.	
	4	Confusing how far is a data value from the mean with the absolute frequency of such value (especially when it is relatively large).	When considering "variability", it becomes hard to determine the focused value from its vertical protrusion on the distribution.

It is important to highlight that if only the ability to anticipate likely difficulties that students may face in solving the given task is considered, then Indicator D1 is met by all the teachers in this case study. Moreover, if only the ability to provide the most likely reasons for likely difficulties that students may face in solving the given task is considered, then Indicator D2 is met by three out of four of the teachers in this case study—i.e., by Teachers 1, 2 and 4.

5.8 Results and discussion regarding Question (e)

Only two of the surveyed teachers—Teachers 1 and 4—provided an answer to this question. Teacher 1 considered significant Student 3's "viewpoint about whether the data distributions were clumped closer to the median", because "this answer becomes an opportunity to think whether measures of central tendency in a dataset are mere theoretical values, or they are based on real situations". In the case of Teacher 4, he considered significant Student 2's answer, because "when correctly analyzing data, a mistake that can be easily made is to think that the difference between the maximum and minimum values of the frequency is a measure that accounts for variability".

In these answers, it is possible to identify two distinctive aspects of HCK. In the case of Teacher 1, he appears to perceive significance in what Student 3 is saying because he notices a didactical opportunity in that comment. In the case of Teacher 4, he catches in Student 2's answer a mathematical distortion common among students, a possible precursor to later misinterpretation of variability in histograms. Hence, it seems that only Teachers 1 and 4 met Indicator C1, since they are the only ones in this case study who evidenced ability to correctly identify significance in the answers given by the fictitious students introduced before.

5.9 Results and discussion regarding Question (f)

Teachers must know at what grades particular topics are typically taught. This is a feature of the so-called *knowledge of content and curriculum* (KCC), defined as knowledge about "topics and issues that have been and will be taught in the same subject area in the preceding and later years" (Ball *et al.*, 2008, p. 391). Question (f) seeks to elicit evidence of the KCC held by the respondents, in relation to the statistical ideas present in the task posed in Item 1.

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TABLE 7 – *Translation of the answers to Question (f) given by the teachers in this case study*

		ANSWER GIVEN TO QUESTION (F)
TEACHER	1	Methods to identify trends in data, and the proper way in which they should be used, depending on the situation.
	2	Elementary school: to make judgments from characteristics of the graphs, such as shape, etc. Junior high school: to make judgments from the difference between the maximum and the minimum value of the data. Senior high school: to make judgments from the interquartile range, the variance, the standard deviation, and so on.
	3	N.A.
	4	Senior high school: being able to correctly analyze and understand graphs and data (using the computer).

Only three of the participants—Teachers 1, 2 and 4—answered this question (Table 7). Teachers 1 and 4 gave general answers: Teacher 1 mentioned "methods to identify trends in data" without indicating what specific methods he was referring to, or at what grade level they are to be taught—although in Japan "trends of data" is more suitable to the statistical contents taught at secondary school level—; while Teacher 4 mentioned analysis and understanding of graphs and data, without specifying what kind of graphs or methods of analysis he was referring to. Moreover, Teacher 4 also referred to engagement in practical skills such as using the computer—which is highly recommended in the current Japanese COS for senior high school regarding the strand "Analysis of data" (MEXT, 2009, p. 19–20, 24–26)—, possibly in order to minimize the tedious nature of data processing in the case of large samples. Teacher 2—the only respondent who mentioned specific statistical contents taught at school mathematics—provided a general answer when referring to elementary school.

In this question, only one teacher—Teacher 2—made mention of specific statistical contents taught at school mathematics—particularly at secondary level—, although he did not specify the particular grades in which such contents are taught. Teacher 2 made specific reference to the range in junior high school—a statistical idea introduced at Grade 7—, and to three other measures of variation in senior high school—all of them introduced in Grade 10.

It seems evident that the surveyed teachers are more aware of the statistics-related contents and cognitive skills to be developed at the grade level they are teaching. Moreover, from these answers, it would be fair to say that only Teacher 2 fully met Indicator F1. However, the lack of fulfilment of Indicator F1 by Teachers 1 and 4—whom showed evidence of partial fulfilment of this indicator—does not necessarily mean they are unaware of how statistical concepts that constitute the mathematics curriculum are taught gradewise.

5.10 Results and discussion regarding Question (g)

5.10.1 Regarding Knowledge of Content and Curriculum (KCC)

According to Ponte (2011, p. 300–301), knowledge of the curriculum—including, among others, its purposes—is not only one of the main poles on which the professional knowledge required for teaching statistics may be standing, but also a fundamental aspect related to lesson planning. Thus, the lesson designed by the respondents will be examined for evidence of whether or not statistics-related curriculum goals are understood and supported.

All the participants answered this question (Table 8). It seems that all the respondents, in some degree, supported and knew about the curriculum objectives in relation to the learning of the contents in the strand "Analysis of data". All the teachers planned lessons in which the foci of instruction were the understanding of statistical ideas such as variance and standard deviation, as well as the explanations and argumentation from the students. However, some teachers seem to be more knowledgeable about the curriculum objectives related to the strand "Analysis of data" than others. For example, Teacher 1 focused his lesson on the understanding of multiple statistical ideas—i.e., the range, variance, standard deviation, and interquartile range—, in order to make students use them to grasp the variability of the data and make explanations through argumentation, which is in line with the curriculum goals mentioned in the COS. The case of Teacher 3 is also worthy to be highlighted. He was the only teacher who explicitly pointed out linking variability in data with real events, as well as the only one who suggested the study of variability through computer technology, being both practices two of the remarks made in the COS concerning the strand "Analysis of data".

TABLE 8 – *Translation of the answers to Question (g) given by the teachers in this case study*

TEACHER 1	TEACHER 2
TASK: «Among 2 distributions, which one do you think has more variability?» ACTIVITIES: (a) Check different ways (range, variance, standard deviation, interquartile range) for examining variability. (b) Place students in groups, asking to each group to use only one of the methods in (a) to discuss about what things could be told about the variability of the given distributions. (c) Each group will share with the rest of the classroom what they considered in (b). (d) Depending on the method used, and while checking different considerations, think about how to look at variability. Students will experience personally the need of using several methods and finding out the appropriate one in order to consider data trends.	(a) To make students think about which of 2 given histograms, A and B, has more variability. (b) To make students think about whether they can make their decision based only on the sample size. (c) To judge the variability using the variance. (d) To give practice problems to students.

TEACHER 3	TEACHER 4
In mathematics there are a large number of approaches in many directions concerning "variability": Introduction of the formulas related to variability. Studying variability through the use of computer technology. Based on the aforementioned approaches, bring up for discussion various topics in society and the corporate world, such as product development, among others, as well as their connections with practical applications.	− Give 2 histograms, A and B. − To make students think about in which histogram the variability is larger, and to make them expose about what they think. …At this stage, a detailed explanation about "variability" has not yet been provided. − After their presentations, to explain about "variability", and to make students think again about which histogram has more variability. − To explain, among other things, different terms besides "variability", provide different histograms, and practice.

It is worthwhile noticing that, despite the explicit mention in the Japanese COS of boxplots as useful tools to explore the degree of variability in data when comparing multiple distributions (MEXT, 2009, p. 25), none of the teachers in this case study made use of such graph to that purpose, which could be interpreted as an inadequate support of the curriculum goals and intentions regarding the teaching and learning of boxplots.

5.10.2 Regarding Knowledge of Content and Teaching (KCT)

According to Ball *et al.* (2008), KCT could be evidenced through knowing about different instructionally viable models for teaching a particular idea, as well as knowing how

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to deploy them effectively. Question (g) was also posed in order to elicit evidence of these two indicators associated to KCT—which are identified as E1 and E2 in Table 1. In order to determine the presence of such indicators in teachers' answers, a criterion-referenced assessment rubric was designed, based on the characteristics of effective classroom activities to promote students' understanding of variability compiled by Garfield and Ben-Zvi (2008).

All the four teachers answered this question (Table 8). In relation to Indicator E1, the answers given by Teacher 1 and Teacher 4 are the ones that seem to exhibit a higher level of knowledge on the key characteristics of effective activities that promote students' understanding of variability identified by Garfield and Ben-Zvi (2008), such as the implementation of tasks involving comparisons of data sets, aiming towards describing and representing variability with numerical measures when looking at the given data, and promoting whole-class discussions on how measures of central tendency and variation are revealed in data sets or graphical representations of data (ibid., p. 207–209).

Regarding Indicator E2, the answers given by Teacher 1 and Teacher 4 are also the ones that seem to evidence more knowledge on how to sequence activities and strategies intended to promote students' understanding of variability. For example, in both answers is explicitly stated that the lesson must start by presenting students with some simple data, in order to interpret it (Garfield and Ben-Zvi, 2008, p. 135–137). However, Teacher 4's answer is at an even higher level compared to the others, since explicitly states that variability should be described and compared informally at first—e.g., by describing verbally how the data is spread out—, and then formally, through measures of variation (cf. ibid., p. 208).

Regarding the statistics-related beliefs held by the surveyed teachers, the answers given by three of them—Teachers 1, 2 and 4—provide evidence that they might be *traditionalists*—i.e., teachers more concerned about students gaining algorithmic skills, and less about context and applications—, with only one teacher—Teacher 3—providing evidence of being an *application preparer*—i.e., a teacher focused on teaching theory and algorithms, so students could use them to solve real-world problems. Moreover, it seems that Teachers 1, 2 and 4 see statistics as a process of inquiry—i.e., as a means of answering questions and solving problems. Also, Teachers 1 and 4 planned lessons in which they encourage students to find their own solutions to statistical problems, while fostering the development of statistical discourse and argumentation in the classroom, which provide evidence that they might believe statistics learning to be active learning. The answers given by Teachers 2 and 3 give evidence they might believe that statistics learning is a teacher-centered individual work.

6 Conclusions

Through this study, the author attempts to contribute to the statistics education literature by proposing an eightfold framework for *statistical knowledge for teaching* (SKT), comprised by two facets: one cognitive—comprised of all the six components of mathematics teachers' professional knowledge identified by Ball *et al.* (2008)—and one affective—made up of teachers' conceptions of variability and statistics-related beliefs. By considering these eight traits, specific gaps in statistics education literature are addressed, since none of the previous MKT-based frameworks of SKT reported to date takes into account either all the six components identified by Ball *et al.* (2008), or the role of beliefs and conceptions of variability held by teachers. In addition, a survey instrument, comprised of a task and seven open-ended questions, was developed based on the framework proposed here. Through this

assessment tool, it was possible to gather valuable qualitative evidence about each of the eight hypothesized traits related to SKT from a case study of four Japanese male senior high school mathematics teachers. This not only supported the content validity of the questionnaire, but also made possible to thoroughly examine important aspects of the professional competence to teach statistics held by participants. For example, the use of open-ended questions made possible to identify a teacher holding a conception of variability that was not included in the categorization proposed by Shaughnessy (2007)—i.e., "Variability as visual cues in the graph". Other aspects identified through the analysis of the collected data were the following: weaknesses in the statistical literacy base of the surveyed teachers—e.g., making errors in the computation of particular measures of variation—; teachers holding misconceptions common to students—i.e., to wrongly think that the distribution that is "less symmetrical" is the one with more variability—; lack of support to particular objectives in the COS, such as linking variability in data with real events, studying variability through computer technology, and using boxplots to explore the degree of variability when comparing multiple distributions; and good ability to anticipate difficulties that students might face when solving the posed task.

In addition, meaningful connections between particular traits comprising SKT were drawn from the data analysis. For example, as for Teacher 2, having poor statistical literacy—as in having a misconception—seems to affect the ability to determine the accuracy of students' solutions, as well as the ability to anticipate students' common correct and incorrect answers and difficulties. A deeper study of these connections could be the focus of future studies.

The analysis of the collected data suggests that Japanese senior high school mathematics teachers could require courses where they could develop particular aspects of their professional competence to teach variability-related ideas. Moreover, it would be also possible to incorporate a formative stage in a future implementation of this study, in which teachers receive feedback on their answers and information on strategies that can be used to improve their practice. Thus, future respondents could benefit from participating in this research.

Although a comprehensive picture of the current state of the surveyed teachers' knowledge base on SKT, conceptions of variability, and statistics-related beliefs was obtained through the survey stage of this research, some uncertainties were raised from not being able to interview the respondents and ask them more probing questions. Thus, it could be required to conduct face-to-face interviews in a future study as a follow-up to the questionnaire, in order to explore these traits more deeply and then overcome this methodological problem.

Finally, how representative this case study is of the current situation of Japanese mathematics teachers' professional competence to teach statistics at secondary school is not known. It is offered as a starting point for discussion. Thus, future studies will be needed to reveal the representativeness of this case study.

References

[1]   Ball, D. L., M. Thames, and G. Phelps (2008), Content knowledge for teaching: What makes it special?, *Journal of Teacher Education*, **59**(5), 389–407.

[2]   Batanero, C., and C. Díaz (2010), Training teachers to teach statistics: What can we learn from research?, *Statistique et Enseignement*, **1**(1), 5–20.

O. González

[3] Burgess, T. A. (2011), Teacher knowledge of and for statistical investigations. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education*, Springer, New York, 259–270.

[4] Cooper, L. and F. Shore (2007), Students' reasoning about summary measures from histograms and stem-and-leaf plots. In *American Statistical Association's Proceedings of the Joint Statistical Meetings*, ASA, Alexandria, VA, 2132–2137.

[5] Döhrmann, M., G. Kaiser, and S. Blömeke (2012), The conceptualisation of mathematics competencies in the international teacher education study TEDS-M, *ZDM*, **44**(3), 325–340.

[6] Eichler, A. (2011), Statistics teachers and classroom practices. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education*, Springer, New York, 175–186.

[7] Gal, I. (2004), Statistical literacy. Meanings, components, responsibilities. In D. Ben-Zvi and J. Garfield (Eds.), *The Challenge of Developing Statistical Literacy, Reasoning and Thinking*, Kluwer, Dordrecht, 47–78.

[8] Garfield, J. and D. Ben-Zvi (2008), *Developing students' statistical reasoning: Connecting research and teaching practice*, Springer, New York.

[9] Garfield, J., R. delMas, and B. L. Chance (1999), *Tools for Teaching and Assessing Statistical Inference*. Online: www.tc.umn.edu/~delma001/stat_tools/

[10] Gattuso, L. and M. Ottaviani (2011), Complementing mathematical thinking and statistical thinking in school mathematics. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education*, Springer, New York, 121–132.

[11] González, O. (2012), A framework for assessing statistical knowledge for teaching based on the identification of conceptions of variability held by teachers. In D. Ben-Zvi and K. Makar (Eds.), *Teaching and learning of statistics. Proceedings of Topic Study Group 12, 12th International Congress on Mathematical Education (ICME-12)*, July 8–15, 2012, COEX, Seoul, Korea, 91–100, http://dbz.edtech.haifa.ac.il/publications/books

[12] González, O. and M. Isoda (2011), Variability in education for statistical literacy in mathematics: On the basis of previous survey researches (in Japanese), *Journal of Science Education in Japan*, **35**(2), 139–152.

[13] Groth, R. E. (2007), Toward a conceptualization of statistical knowledge for teaching, *Journal for Research in Mathematics Education*, **38**, 427–437.

[14] Isoda, M. and O. González (2012), Survey on elementary, junior and senior high school teachers' statistical literacy: The need for teacher training in variability (in Japanese). *Journal of Science Education in Japan*, **36**(1), 61–76.

Professional competencies for effective teaching of variability-related ideas: A Japanese case study

[15] Makar, K. and D. Canada (2005), Pre-service teachers' conceptions of variation. In H. Chick and J. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, University of Melbourne, Melbourne, 273–280.

[16] Meletiou-Mavrotheris, M. and C. Lee (2003), Studying the evolution of students' conceptions of variation using the transformative and conjecture-driven research design. In C. Lee (Ed.), *Reasoning about Variability*, Central Michigan University, Mt. Pleasant, MI.

[17] Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT) (2008a), *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics* (in Japanese), Toyokan Publishing, Tokyo.

[18] Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT) (2008b), *Junior High School Teaching Guide for the Japanese Course of Study: Mathematics* (in Japanese), Kyoiku Shuppan Publishers, Tokyo.

[19] Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT) (2009), *Senior High School Teaching Guide for the Japanese Course of Study: Mathematics* (in Japanese), Jikkyo Shuppan Publishers, Tokyo.

[20] National Council of Teachers of Mathematics (NCTM) (2000), *Principles and Standards for School Mathematics*, NCTM, Reston, VA.

[21] Noll, J. A. (2011), Graduate teaching assistants' statistical content knowledge of sampling, *Statistics Education Research Journal*, **10**(2), 48–74.

[22] Peters, S. A. (2009), Conceptions of statistical variation. In S. L. Swars, D. W. Stinson, and S. Lemons-Smith (Eds.), *Proceedings of the 31st annual meeting of the North American Chapter of the IGPME* (Vol. 5), Georgia State University, Atlanta, 644–652.

[23] Petrou, M. and M. Goulding (2011), Conceptualising teachers' mathematical knowledge in teaching. In T. Rowland and K. Ruthven (Eds.), *Mathematical Knowledge in Teaching*, Springer, New York, 9–25.

[24] Pfannkuch, M. and D. Ben-Zvi (2011), Developing teachers' statistical thinking. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education*, Springer, New York, 323–333.

[25] Philipp, R. (2007), Mathematics teachers' beliefs and affect. In F. K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*, Information Age Publications and NCTM, Charlotte, NC, 257–315.

[26] Pierce, R. and H. Chick (2011), Teachers' beliefs about statistics education. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education*, Springer, New York, 151–162.

[27] Ponte, J. P. (2011), Preparing teachers to meet the challenges of statistics education. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School*

O. González

Mathematics-Challenges for Teaching and Teacher Education, Springer, New York, 299–309.

[28] Sánchez, E., C. B. da Silva, and C. Coutinho (2011), Teachers' understanding of variation. In C. Batanero, G. Burrill, and C. Reading (Eds.), *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education*, Springer, New York, 211–221.

[29] Shaughnessy, J. M. (2007), Research on statistics' reasoning and learning. In Frank K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*, Information Age Publications and NCTM, Charlotte, NC, 957–1009.

[30] Tatto, M. T., J. Schwille, S. Senk, L. Ingvarsson, G. Rowley, R. Peck, and M. Reckase (2012), *Policy, practice and readiness to teach primary and secondary mathematics in 17 countries*, IAE, Amsterdam.